Applications of Graphical Models

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Brief CV

- Since 2016, Assistant Prof., EOS-ITC
- 2015-2016, Senior Scientist, CVLD, TU Dresden
- 2012-2015, Postdoc, TNT, Leibniz University Hannover
- 2008-2011, Ph.D, Inst. Photogrammetry, Bonn University
- 2016-2020, Co-Chair ISPRS WG Dynamic Scene Analysis
- Main Research Areas: Photogrammetry, Computer Vision

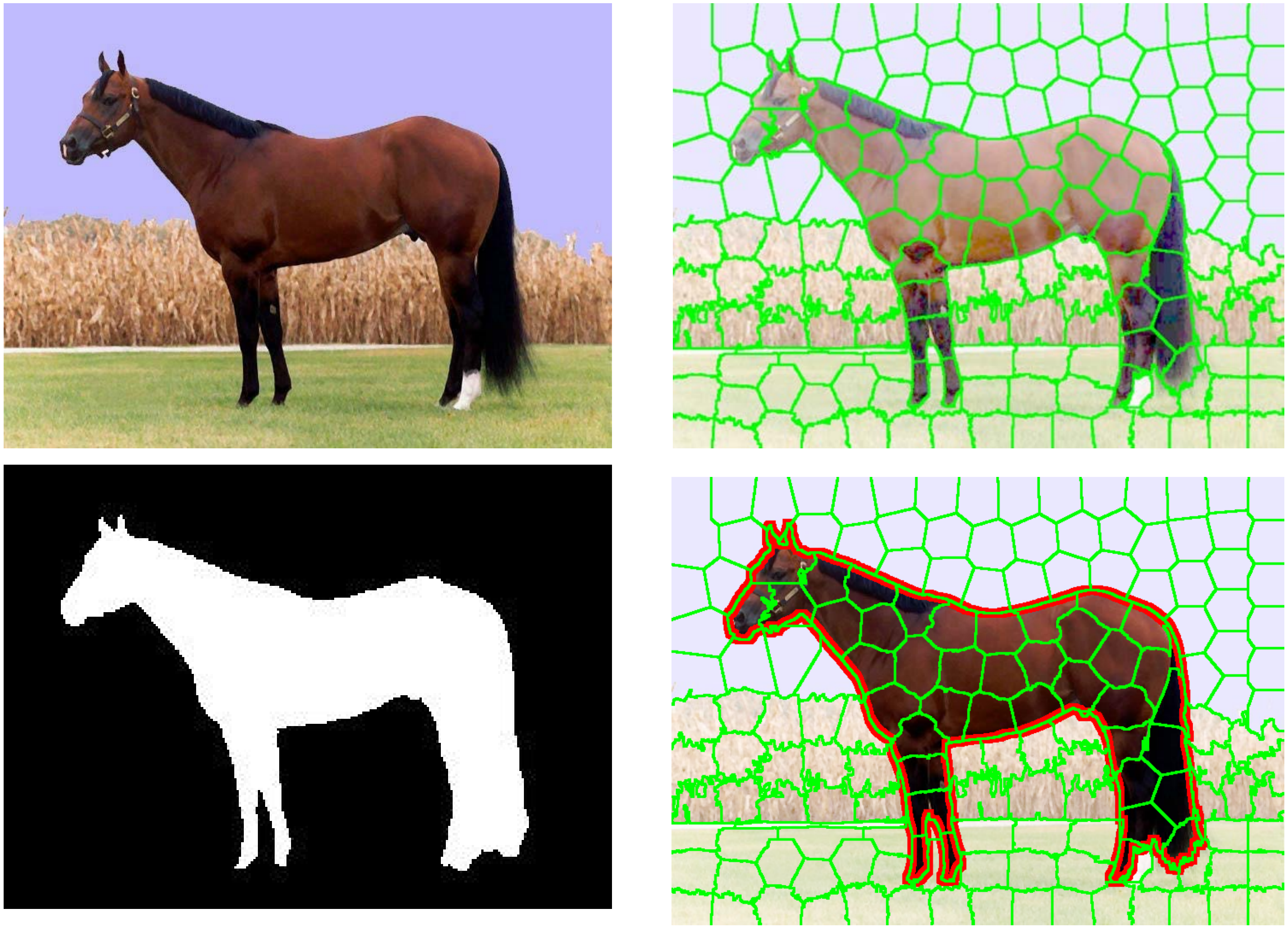
Outline

- **Introduction**
- **Random Fields**
- **Future**

Applications

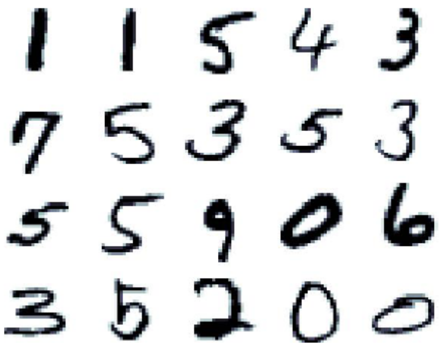
- Medical diagnosis
- Social network models
- Speech recognition
- Robot localization
- Remote sensing
- Natural language processing
- Computer vision
  - Image segmentation
  - Tracking
  - Scene understanding
- Photogrammetry
  - Image classification
  - 3D reconstruction
  - 3D urban modeling
- ....

Segmentation

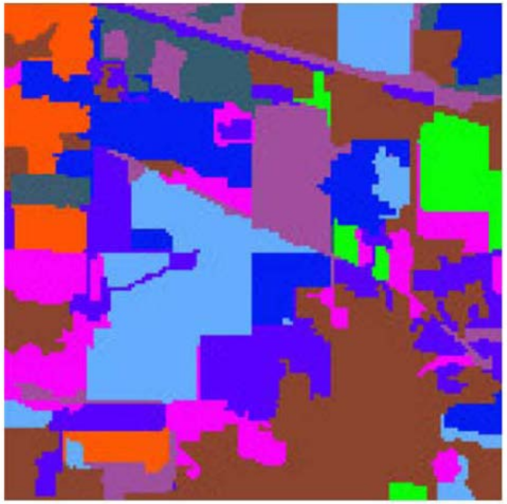


Yang, Rosenhahn, 2016

Classification



(MNIST benchmark data)

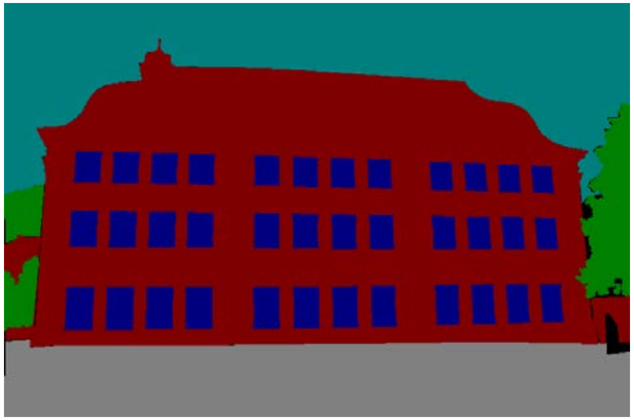


Zhong & Wang 2011

- Reading letters/numbers
- Land-cover classification in remote sensing

Interpretation

Chai et al., 2013

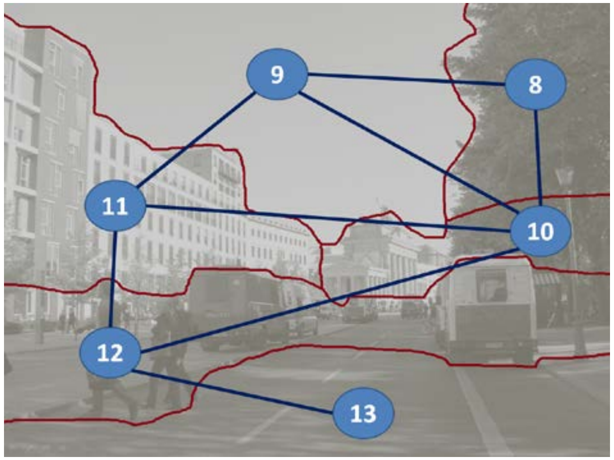


Yang & Förstner, 2011

- Building and road extraction
- Facade interpretation
- Traffic scene interpretation
- Holistic scene analysis

Interpretation

Barth et al., 2010



Yang, 2015

- Building and road extraction
- Facade interpretation
- Traffic scene interpretation
- Holistic scene analysis

Probabilistic Graphical Models

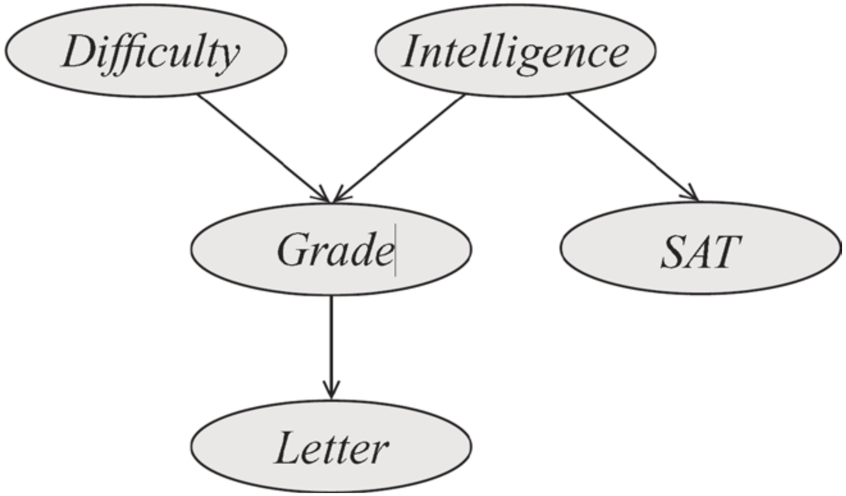
are a marriage between

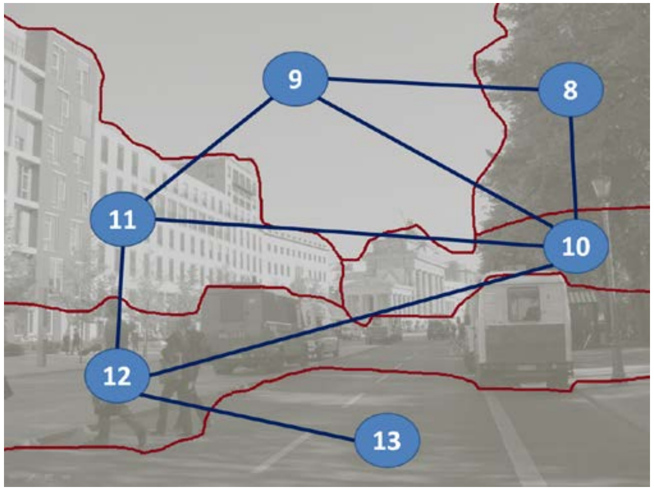
probability theory & graph theory

Graphical Models

Bayesian networks

Conditional/Markov random fields





- **Graph** $\mathcal{G}$

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$$

set of the nodes $\mathcal{V} = \{1, \cdots, i, \cdots, n\}$

set of the undirected edges

$$\mathcal{E} = \{\{i, j\} \mid i, j \in \mathcal{V}\}$$

set of the directed edges

$$\mathcal{A} = \{(i, j) \mid i, j \in \mathcal{V}\}$$

- **Graphical models**

A stochastical model represented by a graph $\mathcal{G}$

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$$

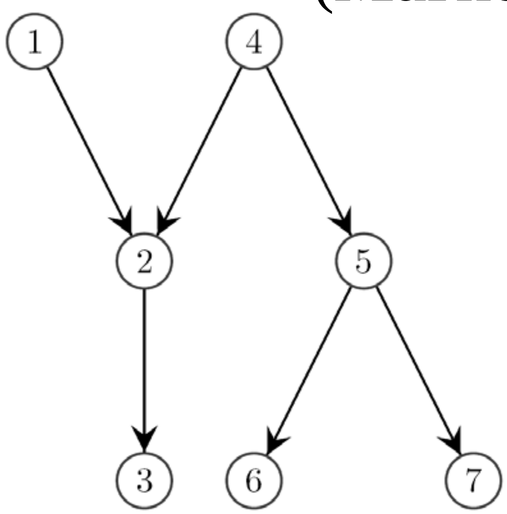
- Nodes $i \in \mathcal{V}$ represent random variables $\underline{\mathbf{x}}_i$
- Edges represent mutual relationships
  - ➢ Undirected edges $\{i, j\}$ model joint probabilities

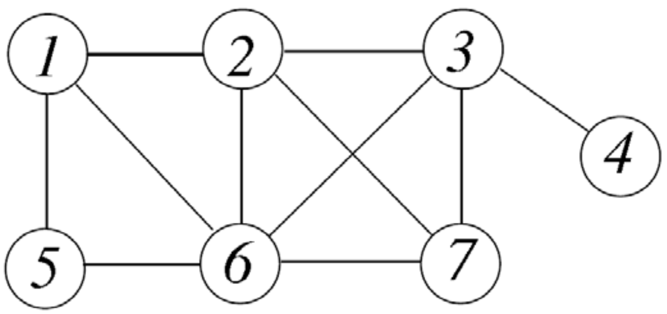
$$P(\mathbf{x}_i, \mathbf{x}_j)$$

  - ➢ Directed edges (i, j) model conditional dependencies

$$P(\mathbf{x}_j \mid \mathbf{x}_i)$$

- **Graphical models**
  - Visualization of dependencies
    - Conditional probabilities : directed edges
      (**Bayesian Networks**)
    - Joint probabilities: undirected edges
      (**Markov Random Field**)





Outline

- Introduction
- **Conditional/Markov Random Fields**
- Future

Applications

Photogrammetry/CV:

- 2D/3D Image Segmentation
- Object Recognition
- 3D Reconstruction
- Stereo / Optical Flow
- Image Denoising
- Texture Synthesis
- Pose Estimation
- Panoramic Stitching
- …

MRFs

- **Definition**

  Markov random field : graphical model over an undirected graph

  + positivity property + Markov property $\mathcal{H} = (\mathcal{V}, \mathcal{E})$

  $P(\mathbf{x}) > 0$

  - Set of random variables linked to nodes

  $$\{\underline{x}_i, i \in \mathcal{V}\} \qquad \underline{\mathbf{x}} = [\underline{x}_i]$$

  - Set of neighbored random variable

  $$\mathcal{N}(x_i) = \{x_j \mid j \in \mathcal{N}_i\}$$

  Markov property:

  $$P(x_i \mid \mathbf{x}_{\mathcal{V}-\{i\}}) = P(x_i \mid \mathbf{x}_{\mathcal{N}_i})$$

MRFs

- **Joint distribution** *(Hammersley & Cliord, 1971)*

  If positive distribution and Markov property:

  Markov random field $\iff$ Gibbs random field

$$P(\mathbf{x}) = \frac{1}{Z} \prod_{c \in \mathcal{C}} \phi_c(\mathbf{x}_c)$$

Potential functions referring to maximal cliques $c \in \mathcal{C}$

$$\phi_c(\mathbf{x}_c) > 0$$

Partition function, normalization constant

$$Z = \sum_{\mathbf{x}} \prod_{c \in \mathcal{C}} \phi_c(\mathbf{x}_c)$$

Sum over *all states* the complete Markov field!

MRFs

- **Equivalent representation of distribution in MRF**

  If positive distribution and Markov property:

  Markov random field $\iff$ Gibbs random field

$$P(\mathbf{x}) = \frac{1}{Z} \prod_{c \in \mathcal{C}} \phi_c(\mathbf{x}_c) = \frac{1}{Z} \exp(-E(\mathbf{x}))$$

Energy

$$E(\mathbf{x}) = \sum_{c \in \mathcal{C}} \varphi_c(\mathbf{x}_c)$$

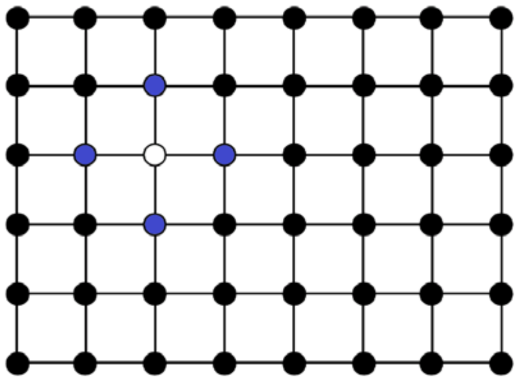
- **Choice of potential functions**

  Need not be probabilities

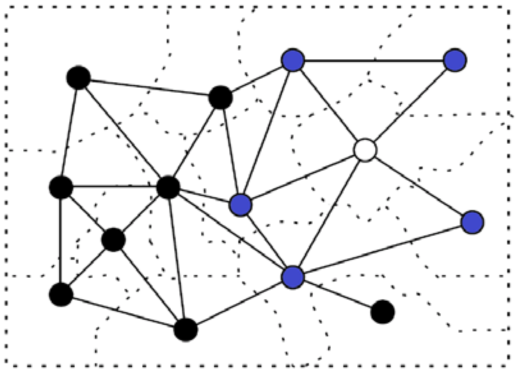
MRFs

- **Structure of MRFs**

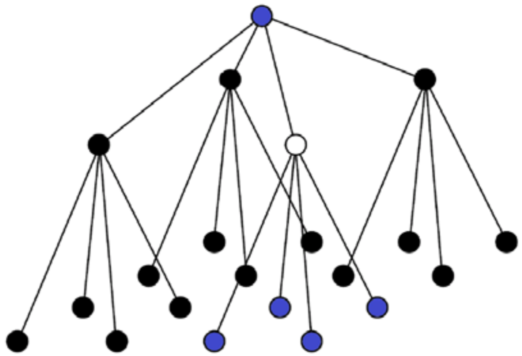
Typical graph structures



rectangular grid



irregular graph



pyramid structure

Figure courtesy of P. Perez

MRFs

- **Pairwise MRFs**

  popular

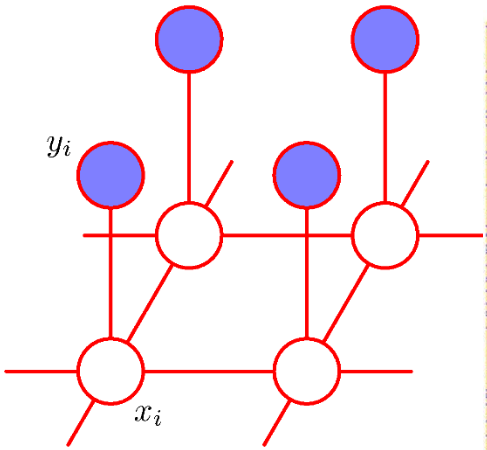
$$P(\mathbf{x}) = \frac{1}{Z}\exp(-E(\mathbf{x}))$$

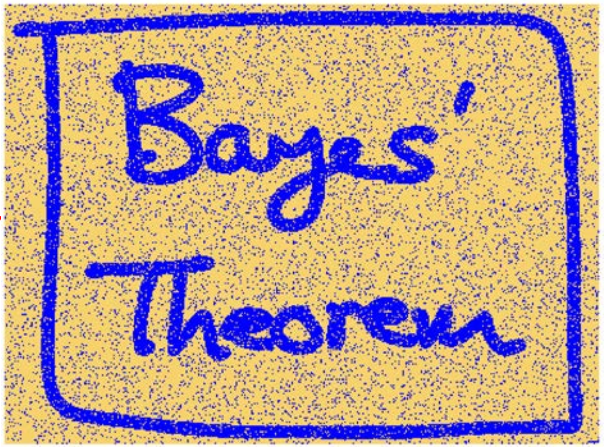
with energy function

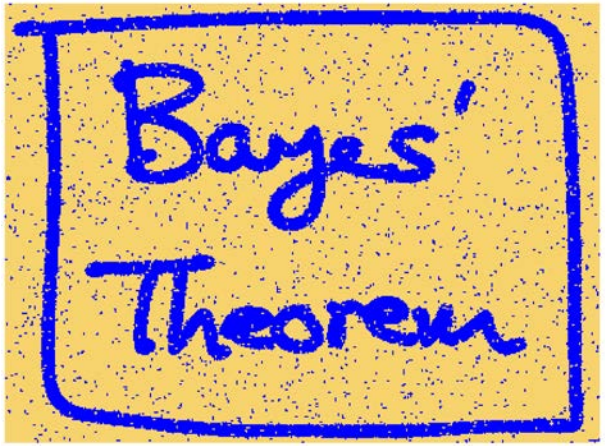
$$E = \sum_{i\in\mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\}\in\mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise}$$

MRFs

- **Image Denoising using Pairwise MRFs**







[From Bishop PRML] noisy image result

CRFs

- **Definition: conditioanl random fields**

A CRF is an MRF globally conditioned on observed data

CRFs

- **Definition: conditioanl random fields**

A CRF is an MRF globally conditioned on observed data

MRF

Joint distribution

$$P(\boldsymbol{x}, \boldsymbol{d}) = \frac{1}{Z} \exp\left(-E(\boldsymbol{x})\right) = \frac{1}{Z} \exp\left(-\sum_{c \in C} \phi_c(\boldsymbol{x}_c)\right)$$

CRF

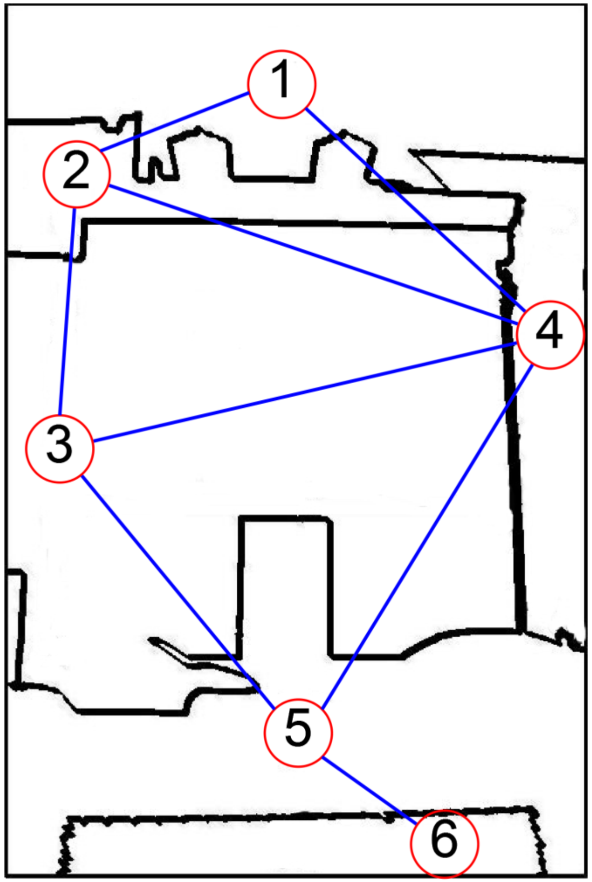
Conditional distribution

$$P(\boldsymbol{x} \mid \boldsymbol{d}) = \frac{1}{Z} \exp\left(-E(\boldsymbol{x} \mid \boldsymbol{d})\right) = \frac{1}{Z} \exp\left(-\sum_{c} \phi_c(\boldsymbol{x}_c \mid \boldsymbol{d})\right)$$

CRFs

Yang & Förstner, 2011

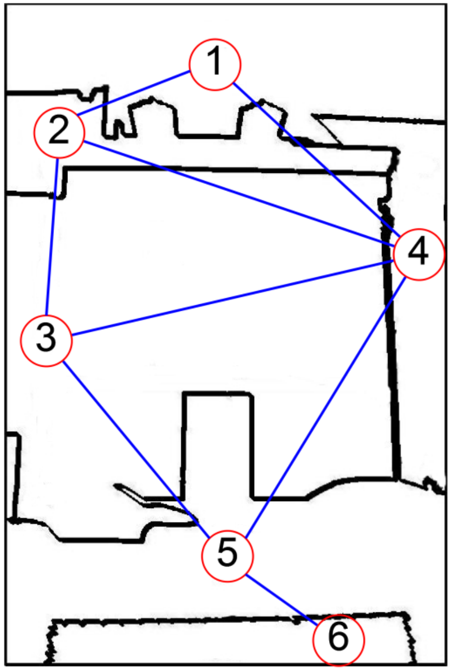
Building facade image



Region adjacency graph

CRFs

CRF has a Gibbs distribution



$$P(\boldsymbol{x} \mid \boldsymbol{d}) = \frac{1}{Z} \exp\left(-E(\boldsymbol{x} \mid \boldsymbol{d})\right)$$

Gibbs energy function (*all dependent on data*)

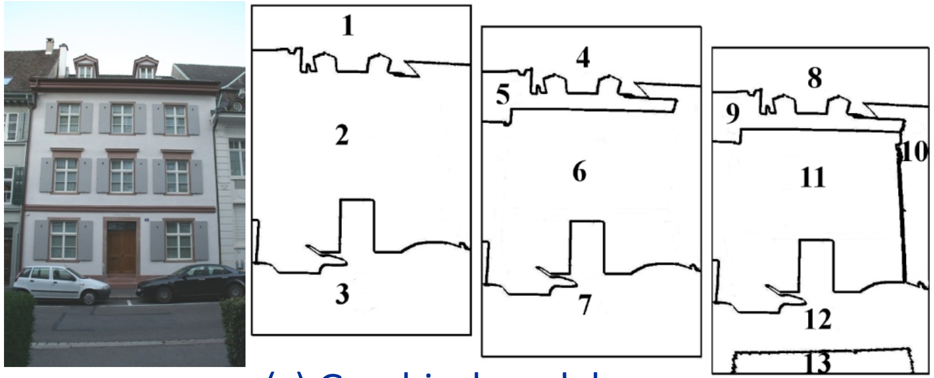
$$E = \sum_{i \in \mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\} \in \mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise}$$

Hierarchical CRFs

Yang & Förstner, 2011

(a) Test image

(b) Multi-scale segmentation



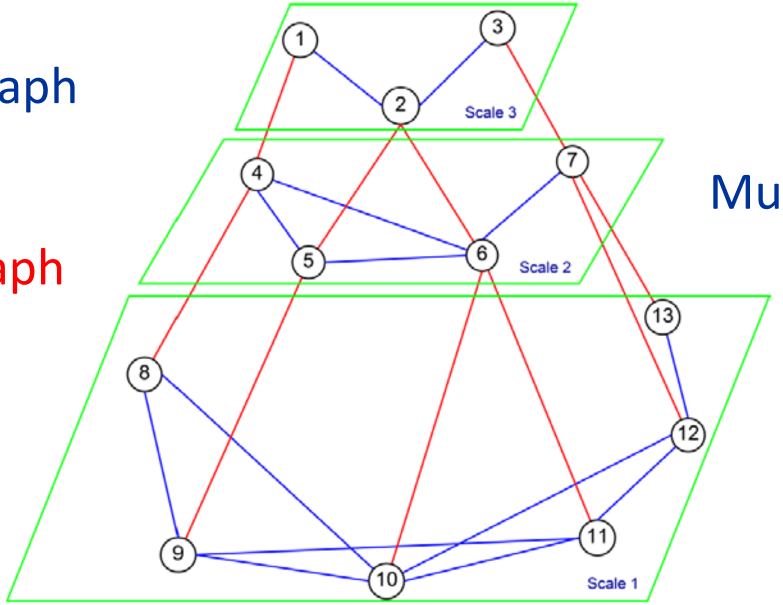
(c) Graphical model

Region adjacency graph

Blue edges

Region hierarchy graph

Red edges



Multi-layer CRF

Hierarchical CRFs

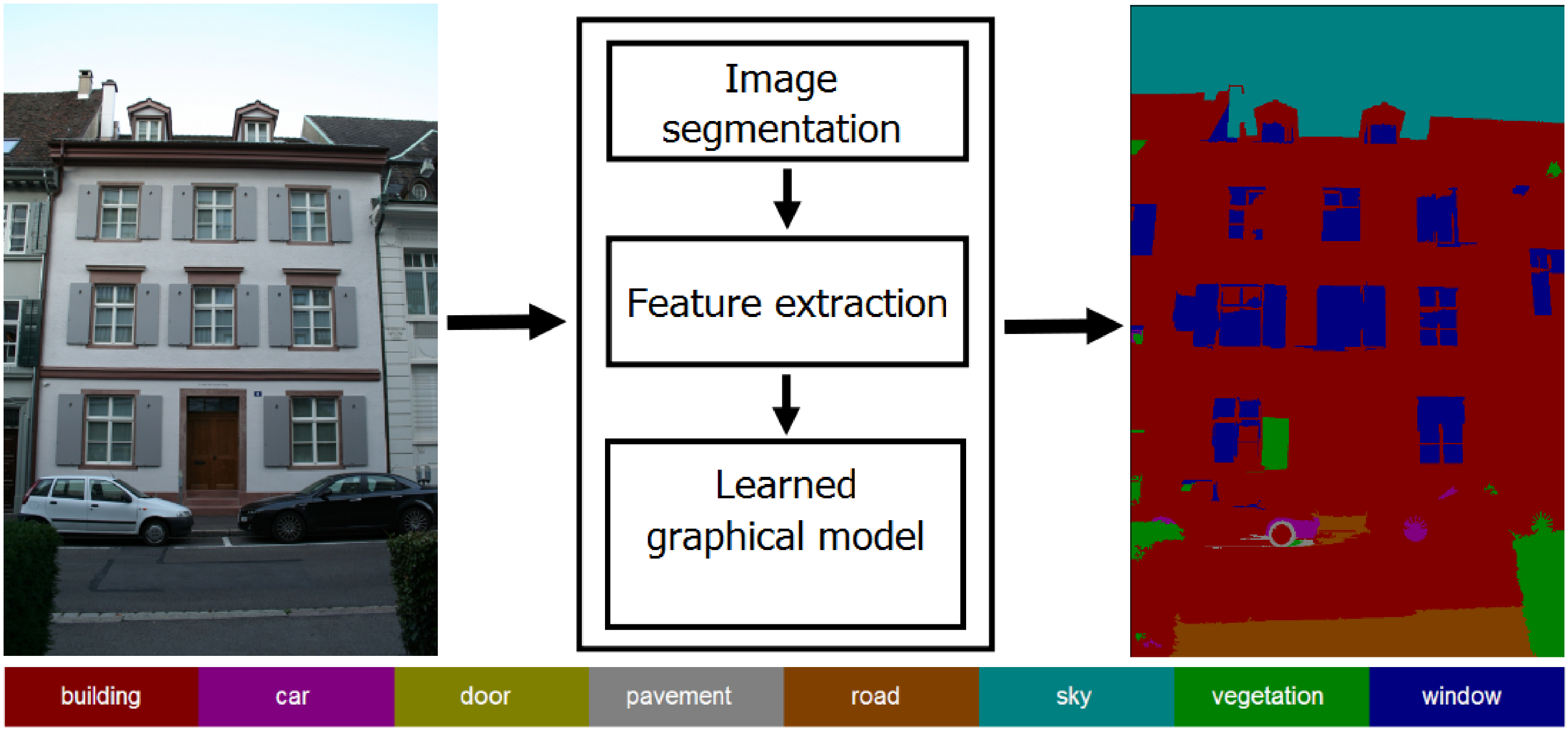
Energy function

$$E = \sum_{i \in \mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\} \in \mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise} + \beta \sum_{\{i,k\} \in \mathcal{H}} \underbrace{E_3(x_i, x_k)}_{Hierarchical}$$

- Unary potential: classifier output (RF)
- Pairwise potential: (Data-dependent) Potts
- Hierarchical potential: (Data-dependent) Potts

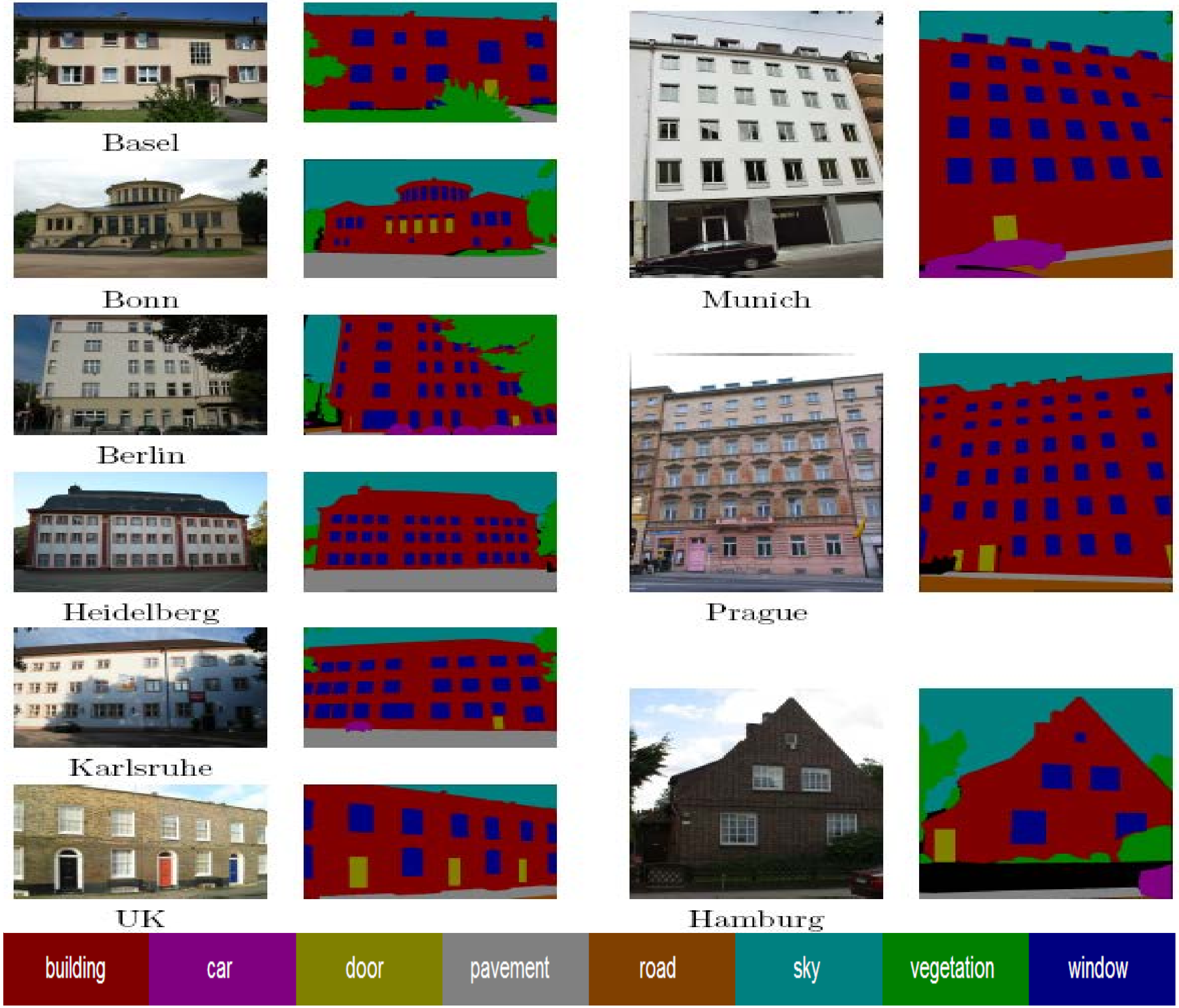
Scene Interpretation

Framework



Workflow for image interpretation of man-made scenes

ETRIMS Database



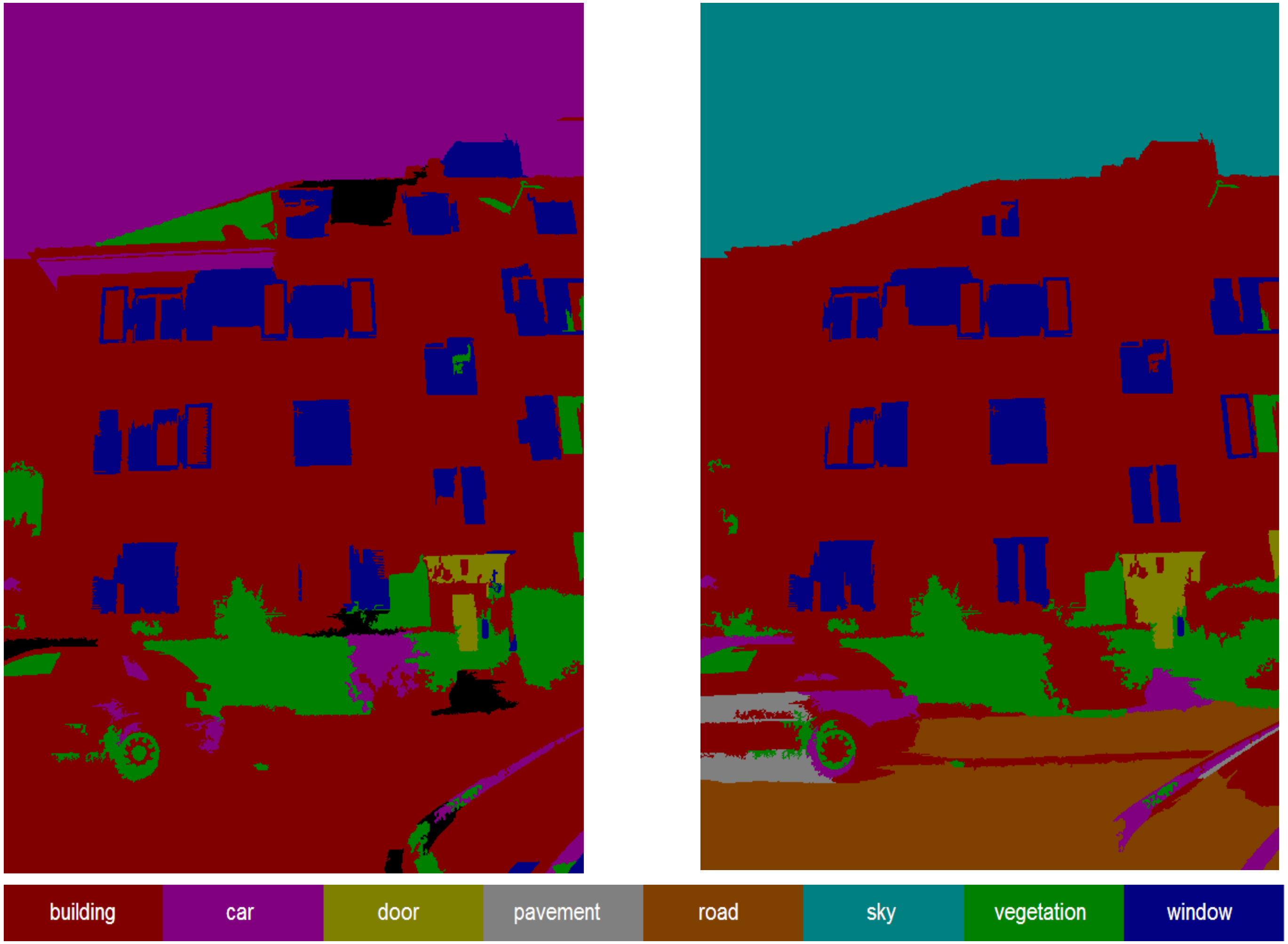
Example Image

building | car | door | pavement | road | sky | vegetation | window

One example image

Ground truth labeling

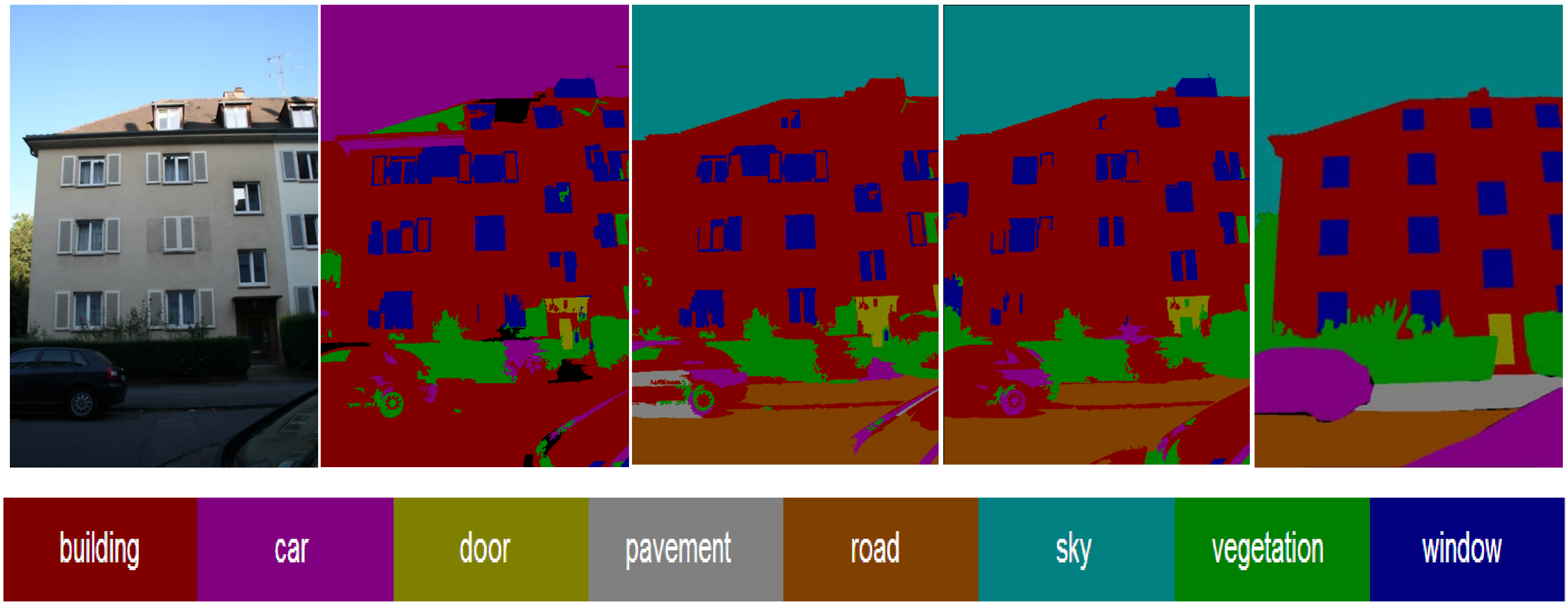
Classification Results



Region classifier (RDF)   Pairwise CRF

HCRF Results

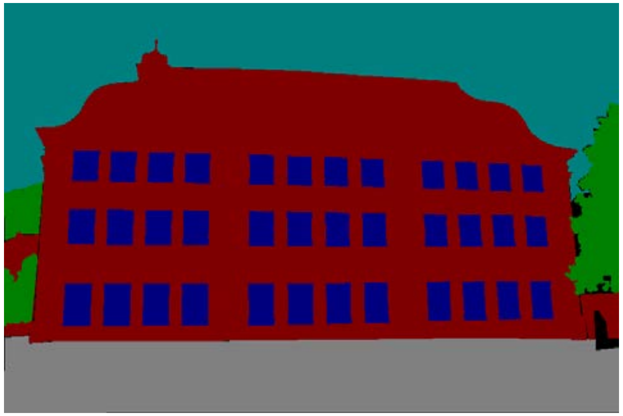
Image | RDF | CRF | HCRF | GT



HCRF Results

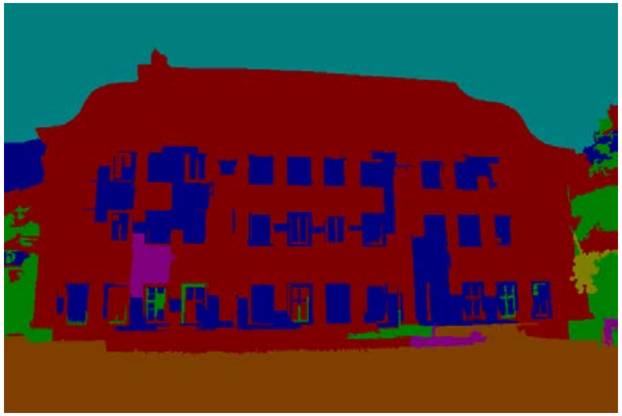
Image

GT

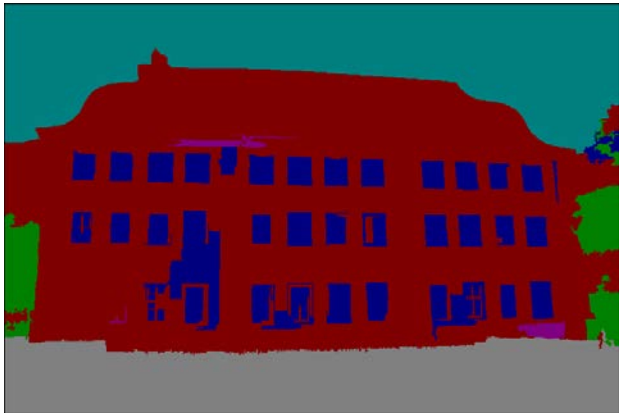


RDF

CRF



HCRF

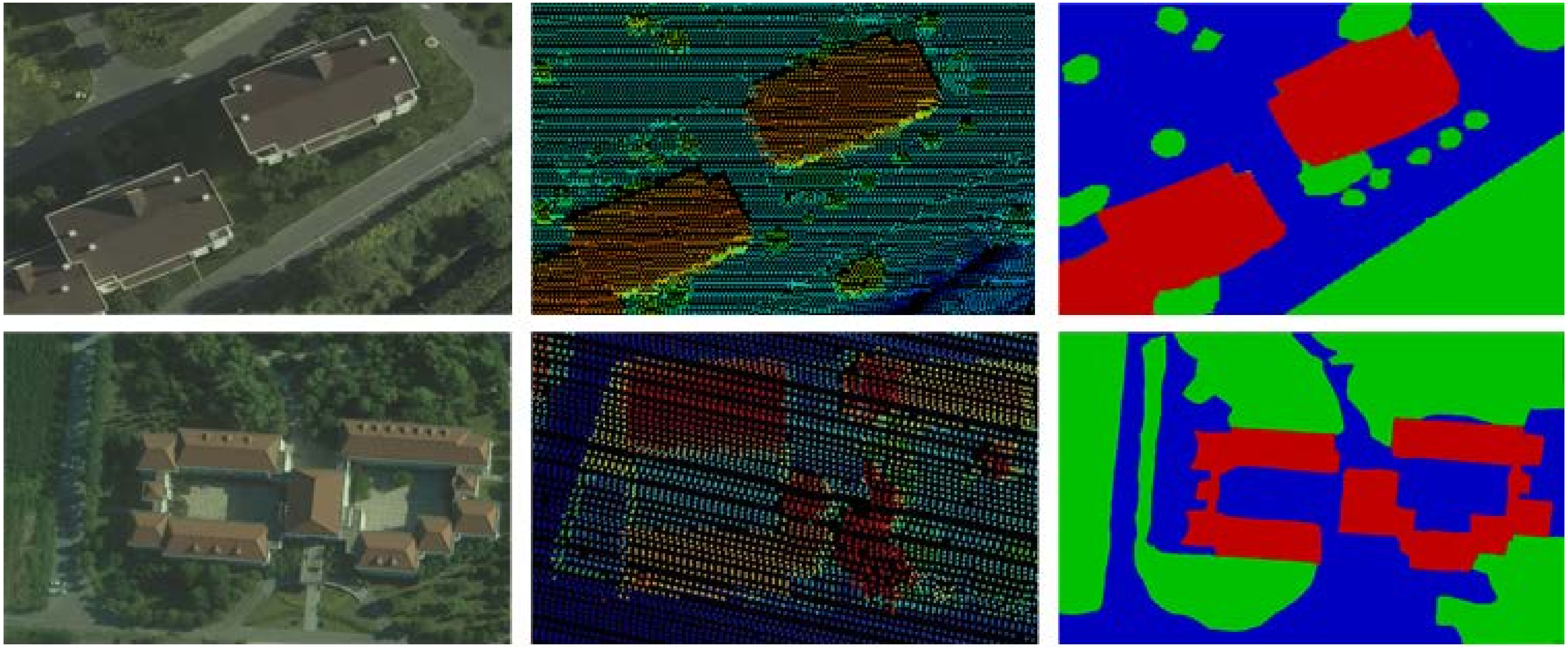


HCRF Results

Pixelwise accuracy comparison

C \ S	watershed	mean shift
RDF	55.4%	58.8%
CRF	61.8%	65.8%
HCRF	65.3%	69.0%

CRF for Sensor Fusion



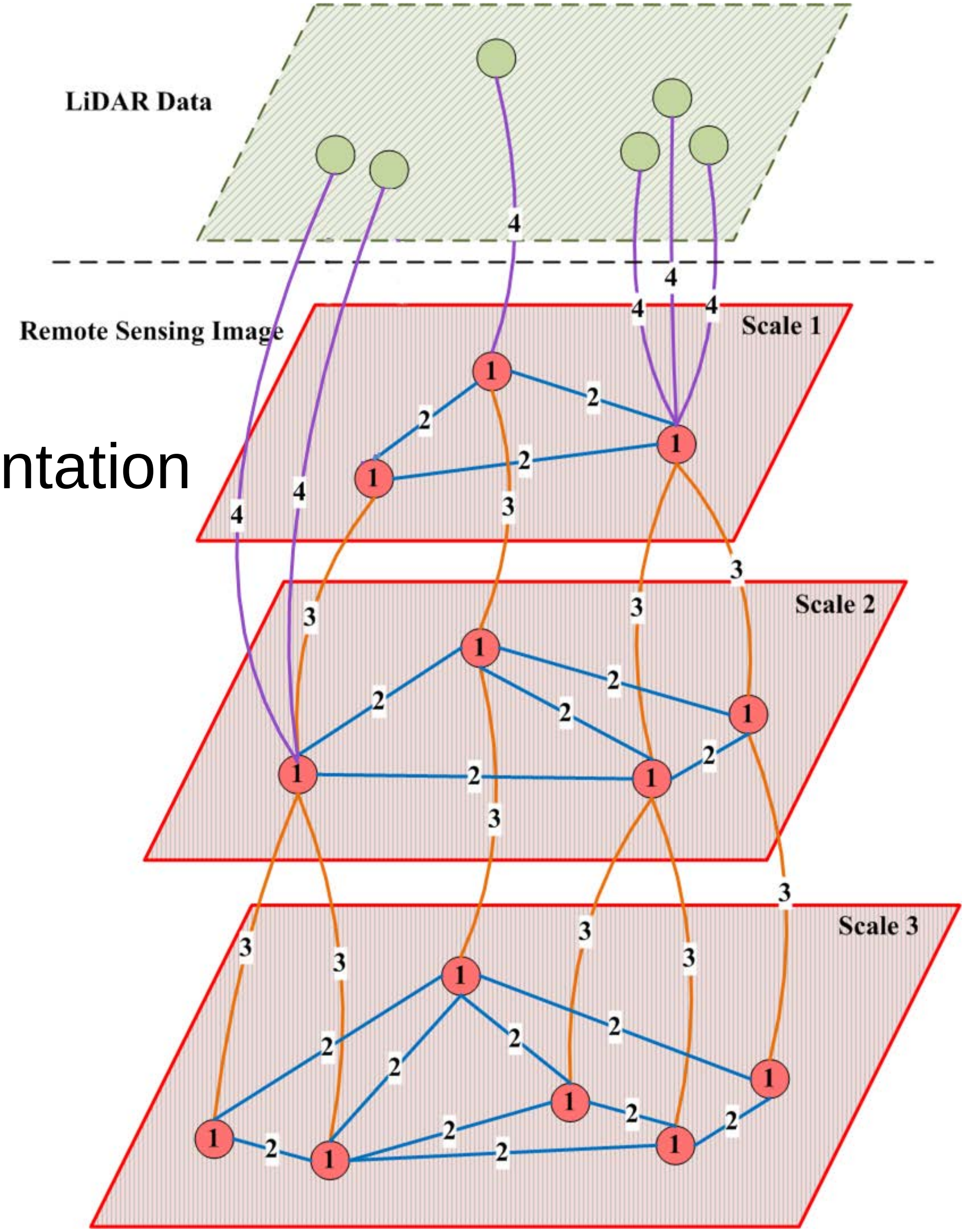
Zhang, Yang, Zhou, 2015

Multi-sensor fusion

- Optical image
- Lidar data

Graph Construction

- ➢ Lidar level
- ➢ Image level
  - ➢ Multi-scale segmentation



MSMSHCRF Model

The conditional probability of the class labels x given an image d and Lidar data L

$$P(\boldsymbol{x} \mid \boldsymbol{d}, \boldsymbol{L}) = \frac{1}{Z} \exp\left(-E(\boldsymbol{x} \mid \boldsymbol{d}, \boldsymbol{L})\right)$$

MSMSHCRF Model

Energy function

$$E = \sum_{i \in \mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\} \in \mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise}$$
$$+ \beta \sum_{\{i,k\} \in \mathcal{S}} \underbrace{E_3(x_i, x_k)}_{scaleHierar.} + \gamma \sum_{\{i,t\} \in \mathcal{M}} \underbrace{E_4(x_i, x_t)}_{sourceHierar.}$$

MSMSHCRF Model

Energy function

$$E = \sum_{i \in \mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\} \in \mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise}$$

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- E1: Unary potentials
  relation between class labels and image
- E2: Pairwise potentials
  relation between class labels of neighboring regions within each scale

MSMSHCRF Model

Energy function

$$E = \sum_{i \in \mathcal{V}} \underbrace{E_1(x_i)}_{Unary} + \alpha \sum_{\{i,j\} \in \mathcal{N}} \underbrace{E_2(x_i, x_j)}_{Pairwise}$$

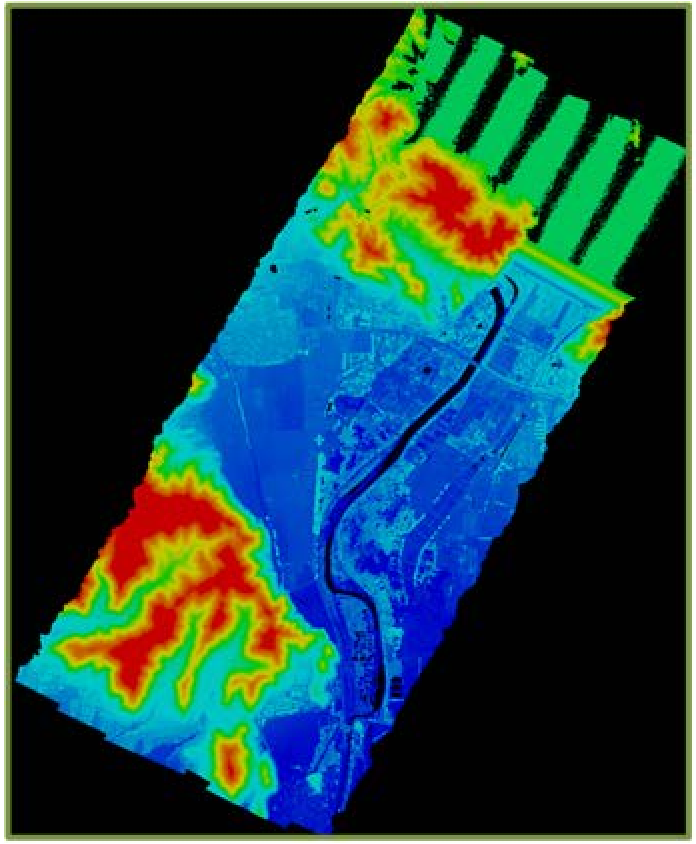
$$+ \beta \sum_{\{i,k\} \in \mathcal{S}} \underbrace{E_3(x_i, x_k)}_{scale Hierar.} + \gamma \sum_{\{i,t\} \in \mathcal{M}} \underbrace{E_4(x_i, x_t)}_{source Hierar.}$$

- E3: Multi-Scale hierarchical pairwise potential
  relation between regions in neighboring scales of images

- E4: Multi-Source hierarchical pairwise potential
  relation between image and Lidar data

Results

Dataset: Beijing Airborne Data



3 classes: {Building, Road, Vegetation}

50 images for training / 50 images for testing

Results

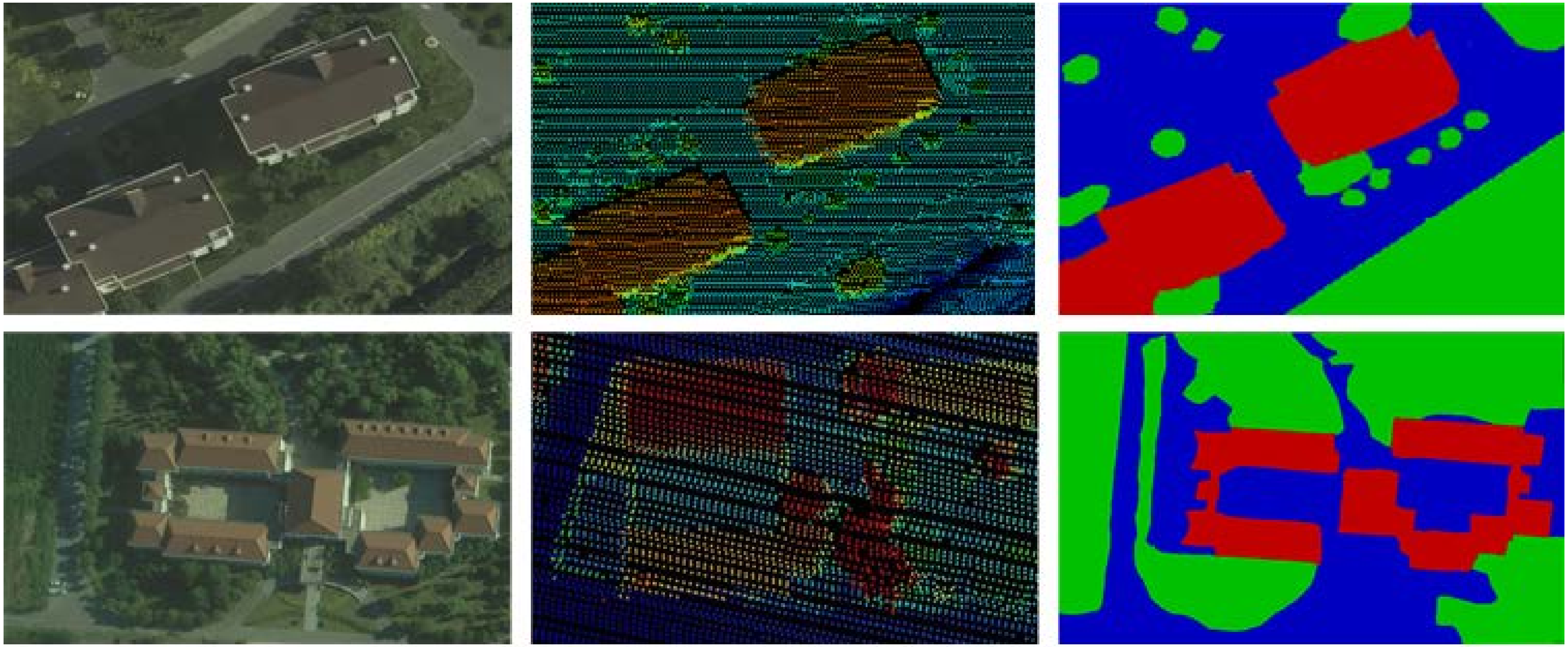


Image Lidar Classification result
(red - building, blue - road, green – vegetation)

Results

Comparison

Method	Accuracy (%)
Standard CRF	64.2
Hierarchical CRF	70.3
Multi-Source CRF	73.6
MSMSCRF	83.7

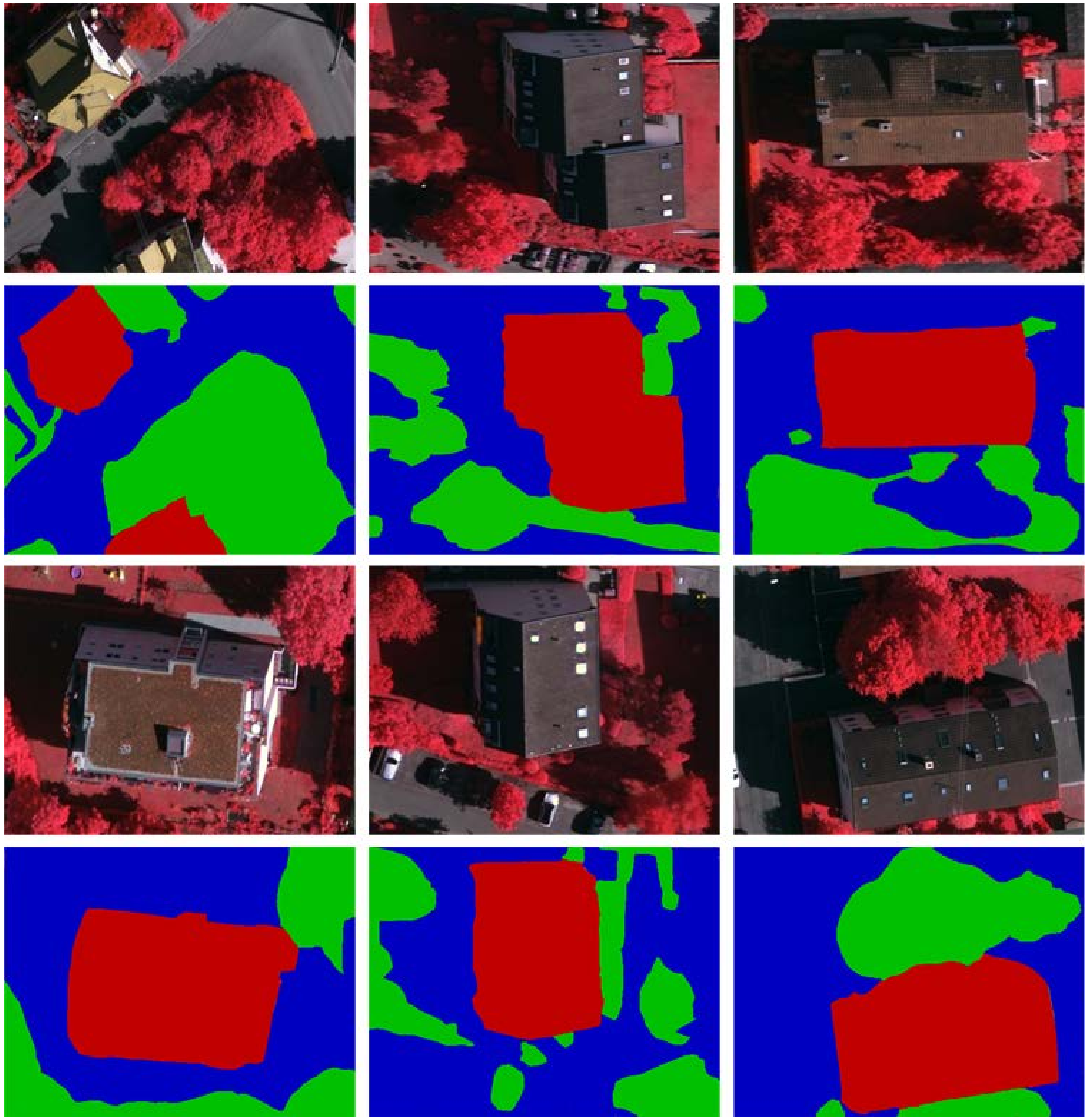
Results

Confusion Matrix

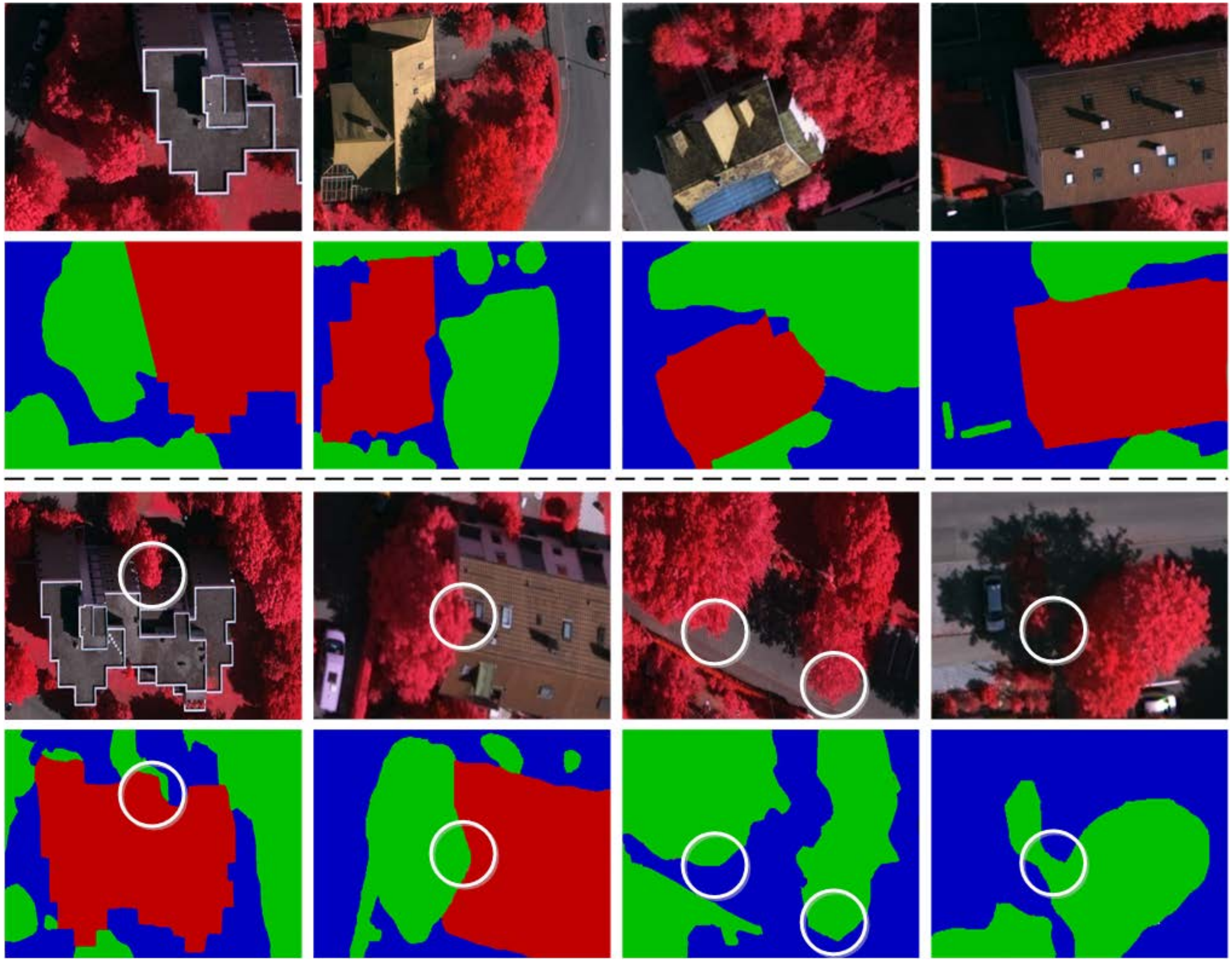
	building	road	vegetation
building	78.3	11.9	9.8
road	9.5	85.9	4.6
vegetation	9.7	8.7	81.6

Results

Dataset: ISPRS Benchmark

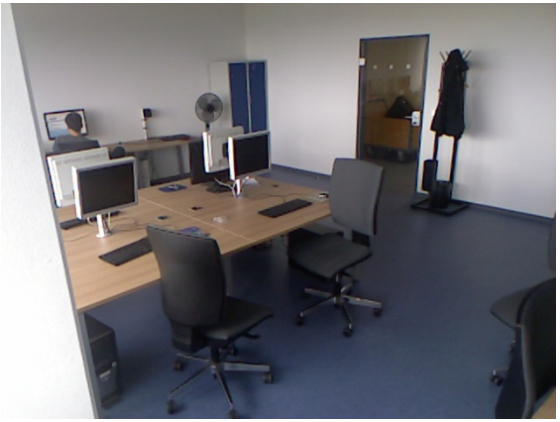


Dataset: ISPRS Benchmark

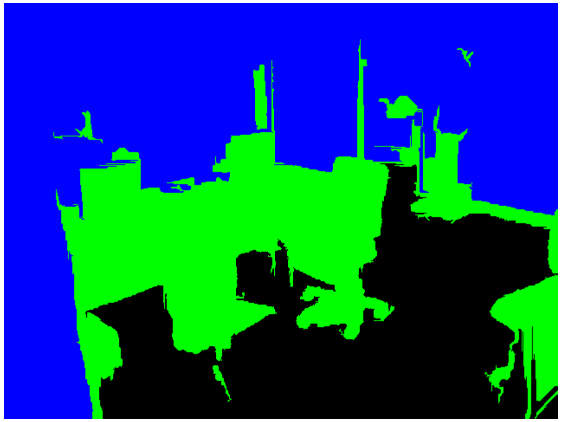


Layout Estimation

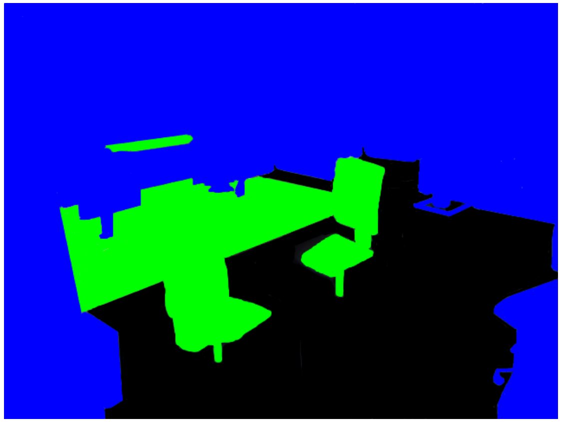
- CRF: fuse RGB and depth



Image+depth



Object/Layout



Ground truth

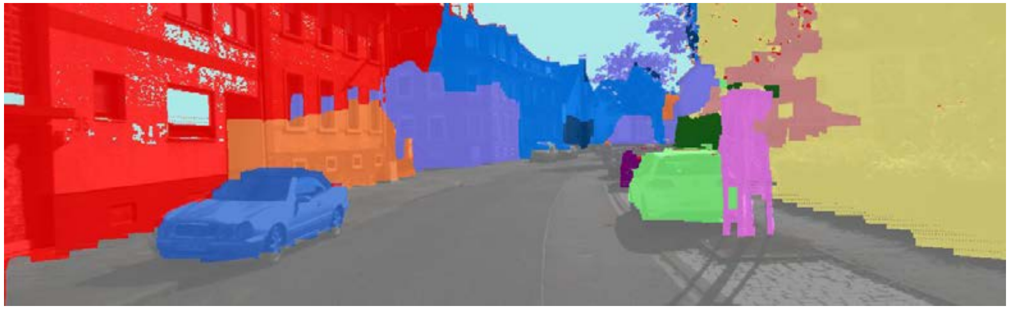
{sitting place, ground floor, background}

- Object/Layout

Shoaib, Yang, Rosenhahn, Ostermann, 2014

Object Segmentation


Single Image

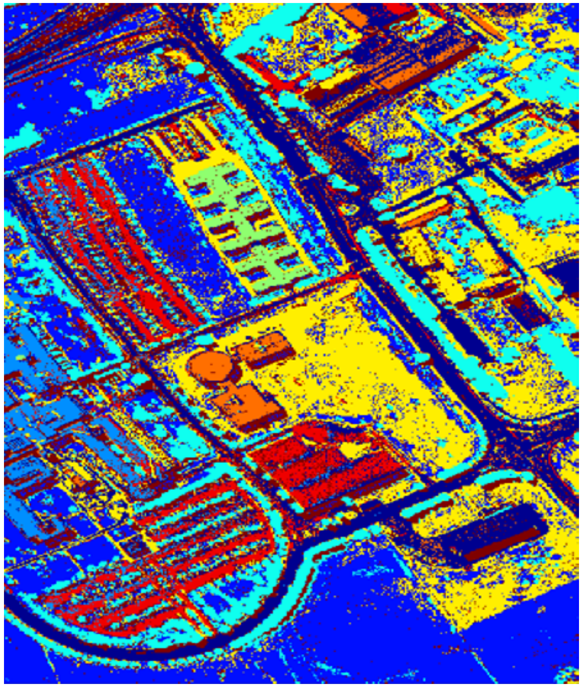

Object Class


Depth Upsampling

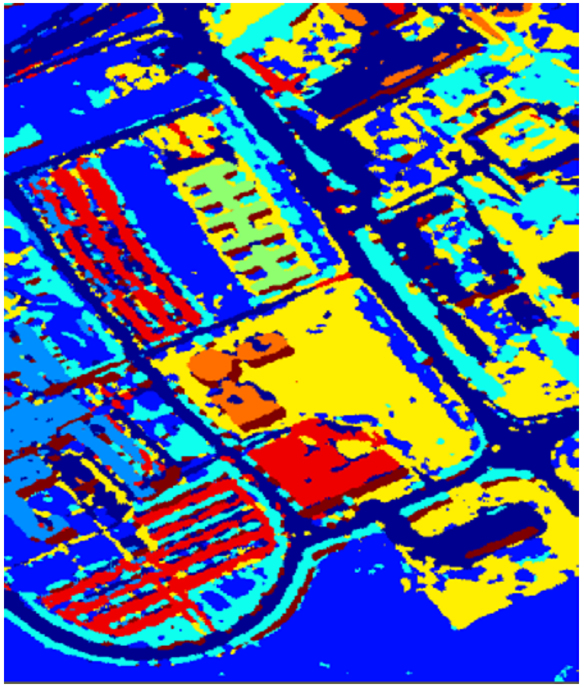
Huang, Gong, Yang, 2015

Hyperspectral Image Classification

Image



GP result



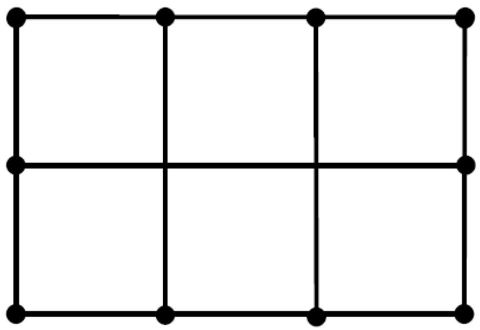
GP-MRF result

Liao, Tang, Rosenhahn, Yang, 2015

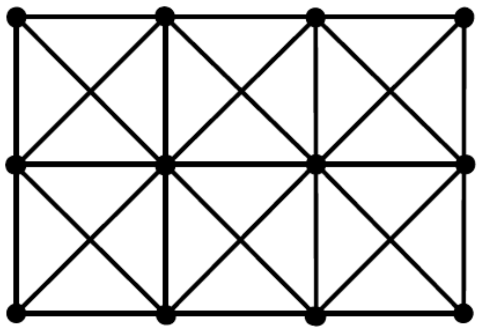
Outline

- Introduction
- Random Fields
- **Future**

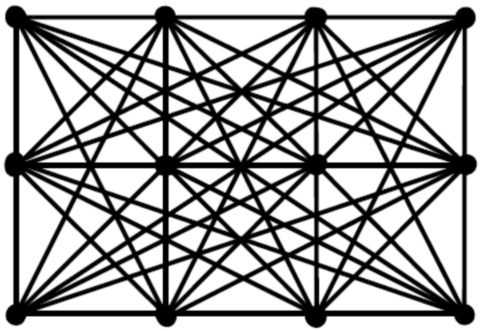
Fully Connected CRF



4-connected CRF



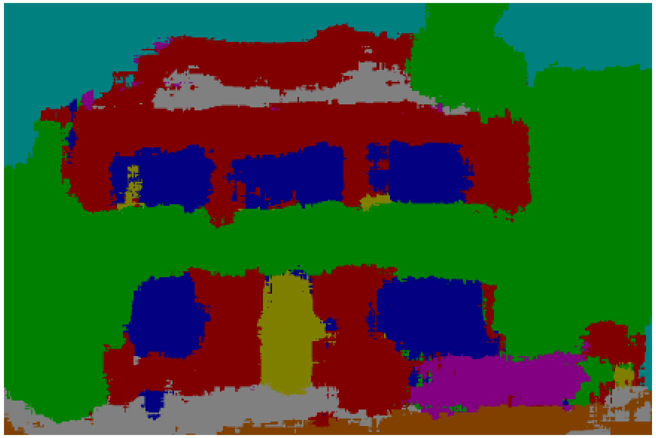
8-connected CRF



Fully-connected CRF

Fully Connected CRF

Image



Unary

Final

Li, Yang, 2016

Fully Connected CRF

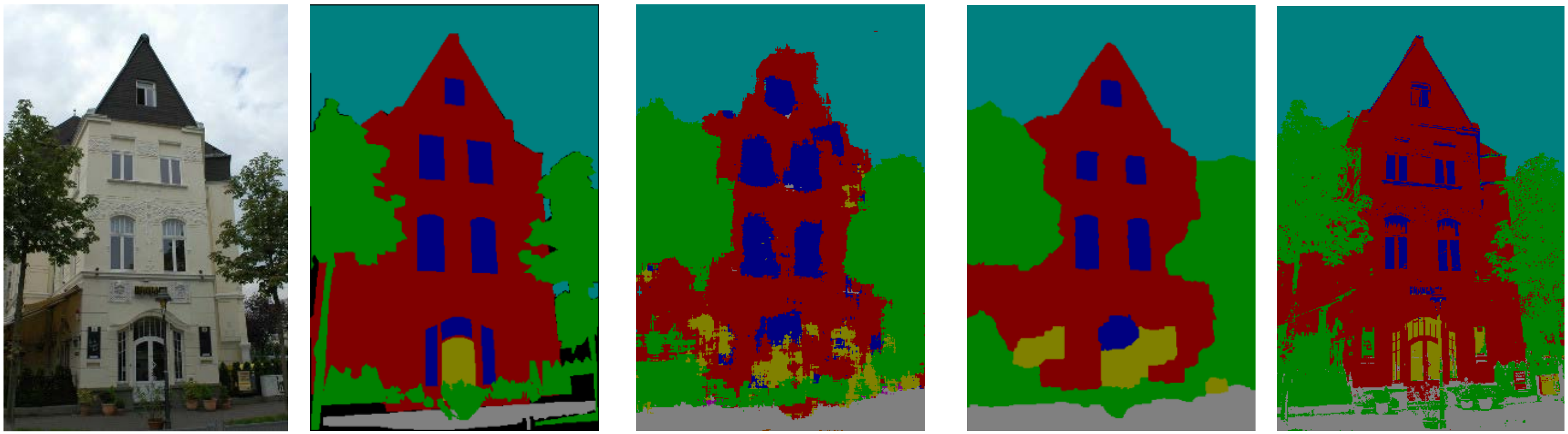
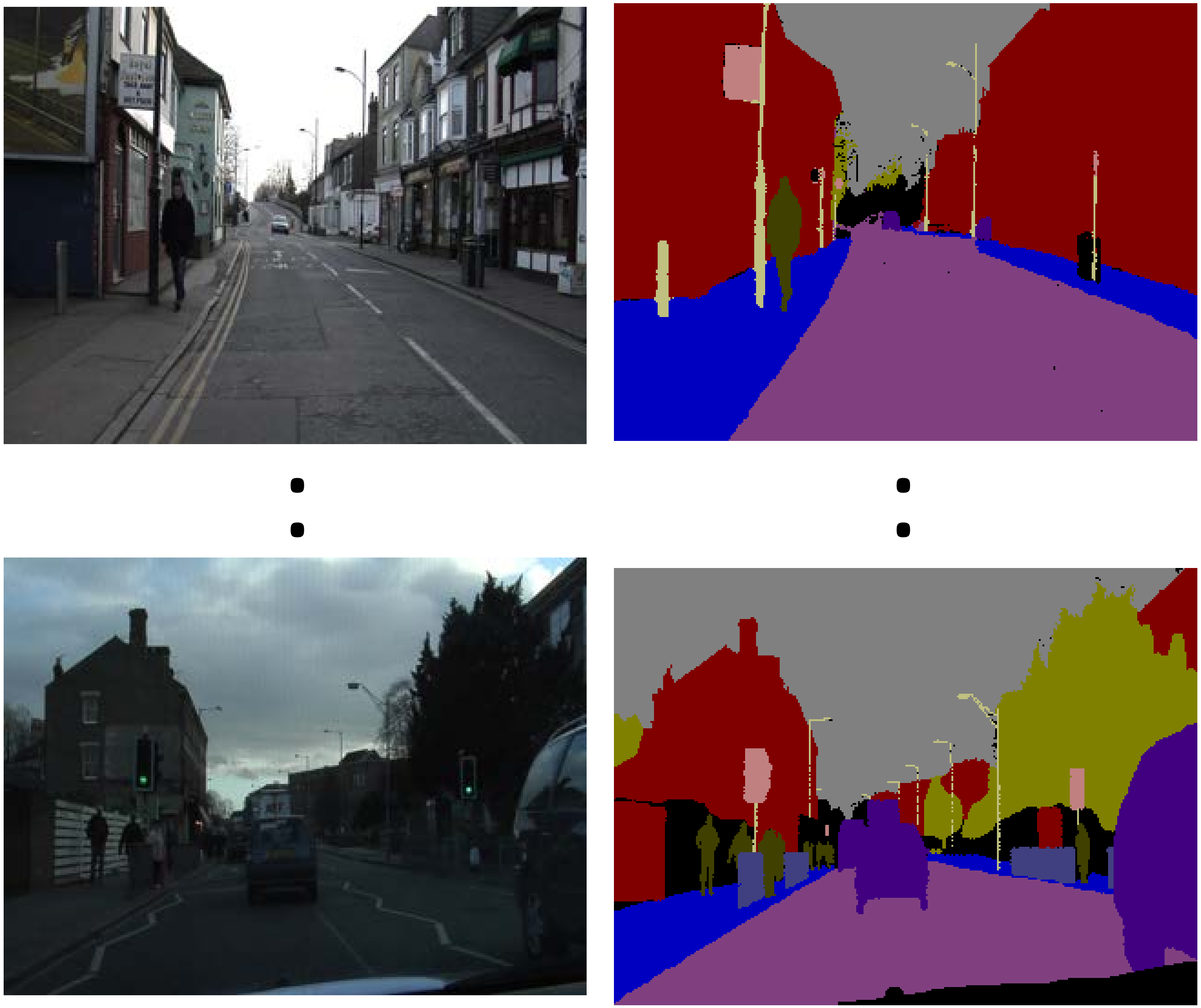
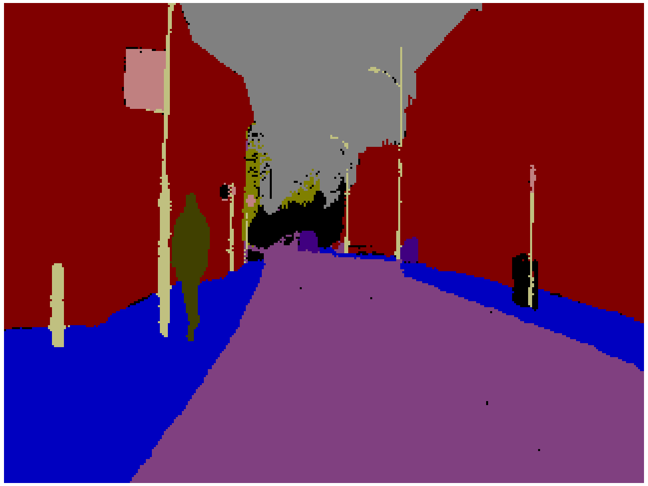


Image GT Texonboost CRF FC-CRF

Semantic Video Segmentation

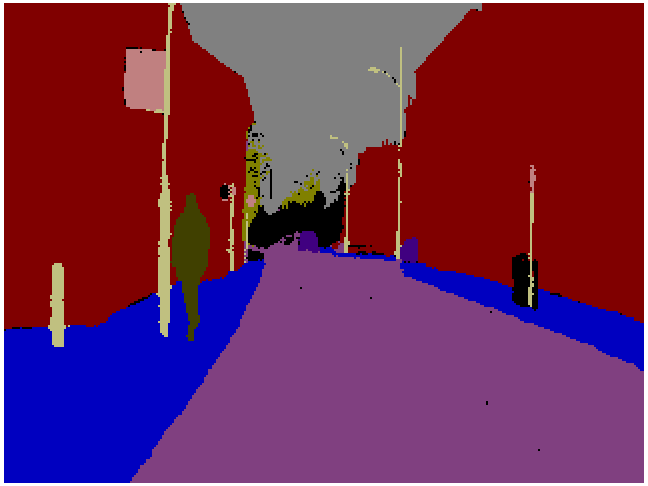


Semantic Video Segmentation



- Spatial-Temporal Deep Structured Models

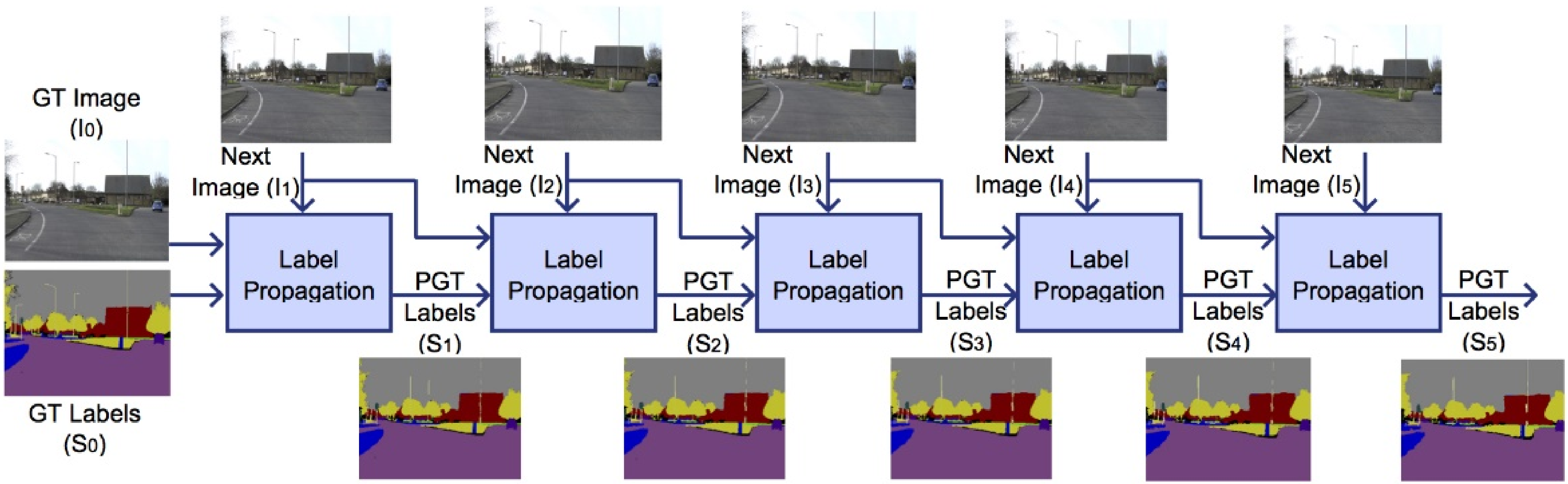
Semantic Video Segmentation

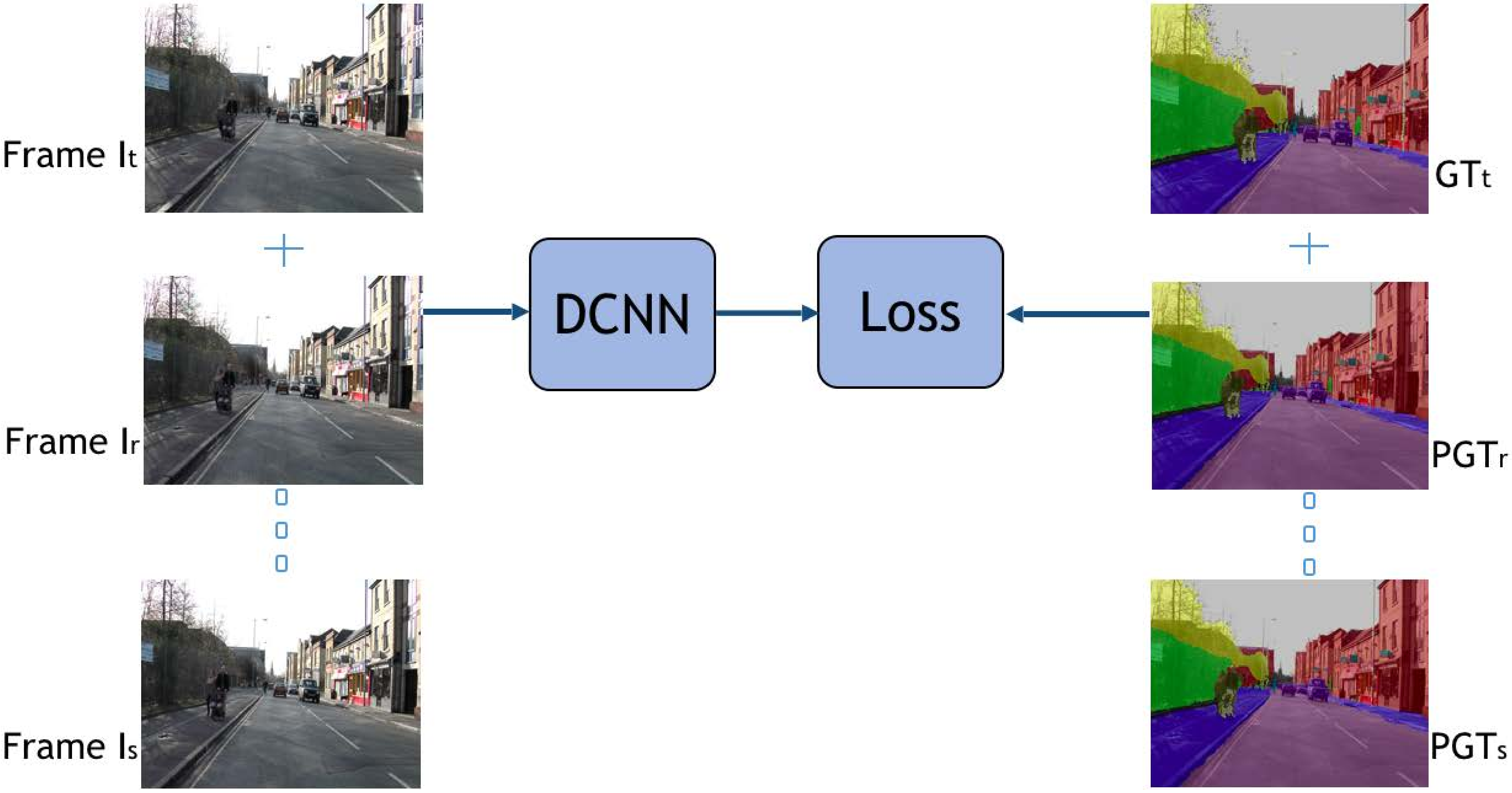
- Spatial-Temporal Deep Structured Models
- Weakly-Supervised Learning CNN+CRF

  - ➢ Basic idea: given a few videos with limited labeled frames, we first estimate pseudo noisy ground truth for each frame in training set. Then we use all the labeled frames to train a CNN.

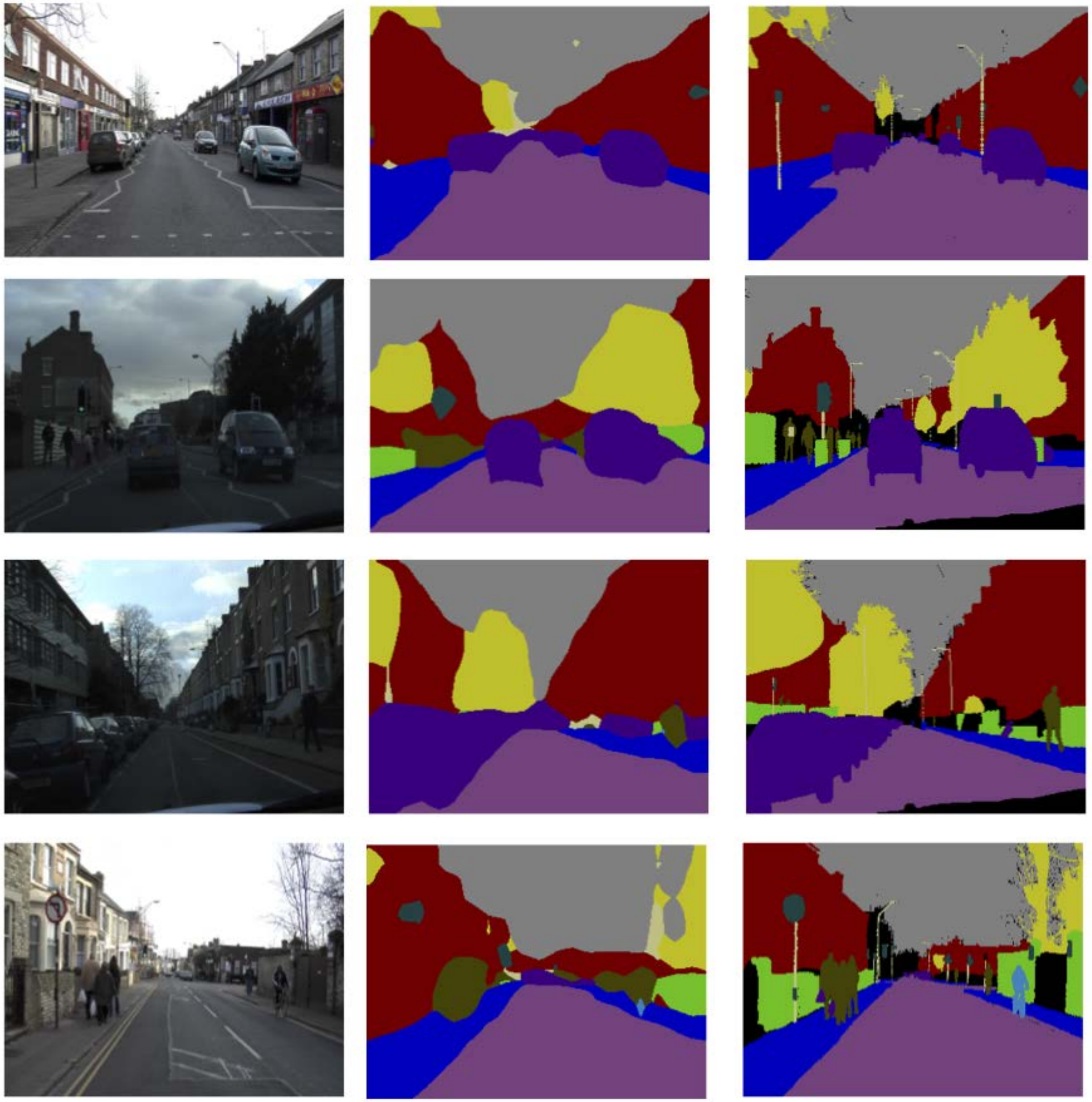
Generating Pseudo Ground Truth Data
CRF for Label Propagation



CNN Training



Results



(a) (b) (c)

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Thank you!

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